

Investigation of a high frequency pulse tube cryocooler driven by a standing wave thermoacoustic engine



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ABSTRACT

In this work, a typical thermoacoustically driven pulse tube cooler as a no-moving part device has been investigated by a numerical method. A standing wave thermoacoustic engine as a prime mover in coupled with an inertance tube pulse tube cryocooler has been modeled. Nonlinear equations of unsteady one-dimensional compressible flow have been solved by the finite volume method. The model presents an important step towards the development of nonlinear simulation tools for the high amplitude thermoacoustic systems that are needed for practical use. The results of the computations show that the self-excited oscillations are well accompanied by the increasing of the pressure amplitude. The necessity of implementation of a nonlinear model to investigate such devices has been proven. The effect of APAT length as an amplifier coupler on the performance of the cooler has been investigated. Furthermore, by using Lagrangian approach, thermodynamic cycle of gas parcels has been attained.

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1. Introduction

Thermoacoustics have been meant as a combination of thermal effects and sound by Rott [1]. He developed the mathematics describing acoustic oscillations in a gas in a channel with an axial temperature gradient, with lateral channel dimensions of the order of the gas thermal penetration which are much shorter than the wavelength. According to the sound wave, thermoacoustic devices can be categorized as standing wave and traveling-wave systems. According to the energy conversion, there are two types of thermoacoustic effects: one is the acoustic oscillation powered by heat energy in thermoacoustic engine (prime mover), and the other is the heat flow driven by acoustic power in thermoacoustic cooler. In thermoacoustic engines, thermal energy from an external source is converted into kinetic energy in the form of acoustic waves. Energy conversion occurs in the stack (or regenerator) which smoothly spans the temperature difference between the hot heat exchanger and the ambient heat exchanger. During the last decade, numerous attempts have been proposed to investigate and to optimize the thermoacoustic engines [2–7]. Parametric studies have been performed for investigation of influences of working liquid [8,9], stack geometry and resonator length [10], resonator diameter

[11], and resonance tube geometry shape [12] on performance of thermoacoustic systems. Furthermore, some general optimizations have been made [13–16].

Linear thermoacoustics has been expounded and summarized systematically by Swift in 1988 [17]. The theory supposes that the time dependence of all fluctuating quantities, such as pressure, velocity, temperature and so on, are purely sinusoidal. It is unable to display the thermodynamic cycle and the mechanism of thermoacoustic conversion clearly. Besides, the linear thermoacoustics is suitable only for stationary thermoacoustic system, and cannot reflect its varying process. It is incapable of dealing with the nonlinear thermoacoustic phenomena, such as amplitude saturation, and frequency shift, which have been observed in lots of experimental thermoacoustic apparatuses.

Nonlinear thermoacoustics as a branch of nonlinear acoustics began to be developed in the last decade. Karpov and Prosperetti from John Hopkins University have done a lot of outstanding work in this field [18–21]. They carried the expansion to fourth order in the perturbation parameter, and obtained explicit results for the initial growth, nonlinear evolution, and final saturation. Then, the dependence of the saturation amplitude upon several important parameters was illustrated. They also investigated the model performance on several examples, including a prime mover, an externally driven thermoacoustic refrigerator, and a combined prime mover/refrigerator system. Both their model and the present model are one-dimensional nonlinear model. However, there are some differences. In the present work, a model based on the finite

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Nomenclature

A	cross-sectional area of gas flow (m^2)
A_V	heat transfer area per unit volume (m^{-1})
C_F	Forchheimer's inertial coefficient
C_P	isobaric specific heat ($\text{J kg}^{-1} \text{K}^{-1}$)
C_S	specific heat of solid ($\text{J kg}^{-1} \text{K}^{-1}$)
C_V	isochoric specific heat ($\text{J kg}^{-1} \text{K}^{-1}$)
d	diameter (m)
d_h	hydraulic diameter (m)
d_g	plate spacing (m)
d_w	wire diameter (m)
F	general function
f	friction factor
j	imaginary unit, $=(-1)^{1/2}$
K	Darcy permeability (m^2)
k	thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)
L	length (m)
l	mesh distance (m)
m	mass (kg)
$\dot{m}$	mass flow rate (kg s^{-1})
Nu	Nusselt number
n	number of screens per inch ($0.0254^{-1} \text{m}^{-1}$)
Pr	Prandtl number
$\dot{q}_{ext}$	external heat load per unit volume (W m^{-3})
R	gas constant ($\text{J kg}^{-1} \text{K}^{-1}$)
Re	Reynolds number
T	temperature (K)
t	time (s)

u	velocity (m s^{-1})
V	volume (m^3)
x	longitude coordinate (m)
Z	compressibility factor
$\Re\{\}$	real part of

Greek letters

α	heat transfer coefficient ($\text{W m}^{-2} \text{K}^{-1}$)
β	opening area ratio of screen
μ	dynamic viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)
ρ	density (kg m^{-3})
ϕ	porosity
ω	angular frequency (rad s^{-1})

Subscripts

I	node number
i	face number
S	solid

Superscripts

$\cdot$ (dot)	time derivative
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Abbreviations

PTC	pulse tube cryocooler
SWE	standing wave engine
SS	stainless steel

volume method with upwind scheme is developed to solve 1D equations of a system with non-perfect gas (via considering a compressibility factor) as the working fluid. In many combined systems such as thermoacoustically-driven pulse tube cooler, high amplitude oscillations occur so that high order harmonics must be considered to model these systems. The present model is not limited by a specified order of approximation.

Thermoacoustically-driven pulse tube cooler (TADPTC) has the advantage of structure simplicity with no moving mechanical components and thus promises a potentially long lifetime. High frequency operation of such a system can lead to a much-reduced size, which is very attractive in small-scale cryogenic applications. Godshalk et al. [22] established the first high frequency TADPTC in 1996. However, the cooler just reached a lowest no-load temperature of 147 K with the frequency being 350 Hz.

Before 2005, pulse tube cooler was directly coupled with the thermoacoustic engine and obtainable pressure ratio for the cooler was limited by the capability of the thermoacoustic engine. In 2005, Dai et al. [23] proposed the concept of acoustic amplifier, which is actually a long tube connecting the engine with the pulse tube cooler. They established and studied a 300 Hz TADPTC utilizing the invention of acoustic pressure amplifier tube (APAT) to couple the standing-wave engine (SWE) with the pulse tube cooler (PTC). APAT is simply a thin circular tube with a length smaller than $1/4$ of wavelength to connect the cooler to the engine. Their theoretical calculation showed that suitable length and diameter of the tube can lead to a pressure wave amplification effect which means that the pressure wave amplitude coming from the thermoacoustic engine can be much amplified to drive the pulse tube cooler.

With modifications, lowest no-load temperatures of 95 K [24], 79.6 K [25], and 77.8 K [26] were sequentially reached. Later, a lowest no-load temperature of 69.5 K has been reached as reported on the work of Zhu et al. [27]. Comparing with the SWE previously used, Zhu et al. made two changes. Firstly, the inner diameters of

the hot heat exchanger, the stack and the ambient heat exchanger were increased to increase inlet volume flow of APAT and the amplification ratio. Secondly, a cone-shaped resonator substituted the isodiametric resonator to decrease the viscous resistance. Zhu et al. [28] have investigated the influences of different parameters such as the engine stack lengths, the geometry of the resonator, the length of the APAT as well as the mean pressure on the overall performance of the device, experimentally. Most recently, Wang et al. [29] obtained a cooling power of 1.04 W at 80 K and a no-load temperature of 63 K with 500 W heating power.

The present work reports on the investigations of a 300 Hz frequency thermoacoustically driven inertance tube pulse tube cryocooler by using a numerical nonlinear model. The developed numerical model is based on nonlinear one-dimensional unsteady compressible flow equations solved by the finite volume method. APAT length as a key design parameter and its effect on the pressure ratio, the hot temperature of SWE, the cooling temperature of PTC, mass flow rate, and pressure amplitude are investigated.

2. Physical model

2.1. Geometry of the device

The geometry of the device including the heat exchangers, stack, resonator, and buffers for SWE, and including the heat exchangers, regenerator, pulse tube, straightener, and inertance tubes, for PTC, and also including APAT (Acoustic Pressure Amplifier Tube) as a coupler, is shown in Fig. 1.

2.2. Governing equations

Because the mass flow, momentum flow, and the enthalpy flow in the longitudinal direction is very greater than those of lateral

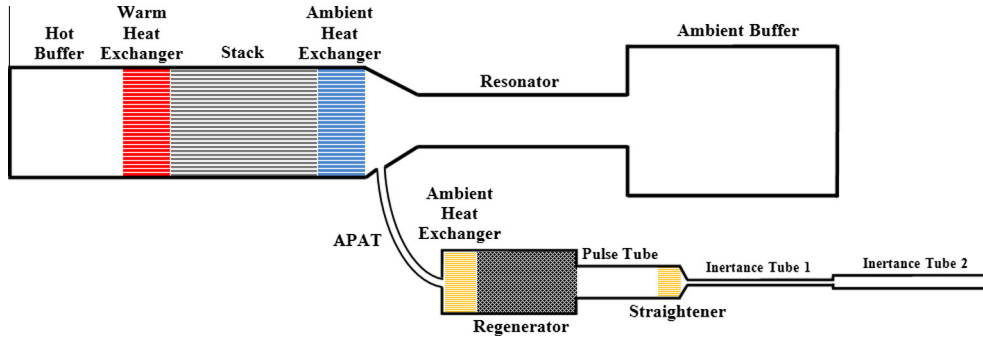


Fig. 1. Schematic of a standing wave thermoacoustic engine coupled with a pulse tube cooler.

directions are, a one-dimensional model can describe the performance of the system, acceptably. Thermal non-equilibrium model for porous parts (stack, HXs, and regenerator) is used. Governing equations include continuity, momentum equation, energy equation, and equation of state for gas, and energy equation for solids of porous parts as follows:

Continuity equation:

$$\frac{\partial}{\partial t}(\rho\phi) + \frac{\partial}{\partial x}(\rho u\phi) = 0 \quad (1)$$

Momentum equation:

$$\frac{\partial}{\partial t}(\rho\phi u) + \frac{\partial}{\partial x}(\rho\phi u^2) = -\phi \frac{\partial P}{\partial x} - \phi \left(\frac{\mu}{K} \phi u + \frac{C_F \rho}{\sqrt{K}} \phi^2 u |u| \right) \quad (2)$$

Energy equation for gas:

$$\begin{aligned} \frac{\partial}{\partial t} \left[\rho\phi \left(C_V T + \frac{u^2}{2} \right) \right] + \frac{\partial}{\partial x} \left[\rho u \phi \left(C_P T + \frac{u^2}{2} \right) \right] \\ = \frac{\partial}{\partial x} \left(k\phi \frac{\partial T}{\partial x} \right) + \alpha A_V (T_s - T) \end{aligned} \quad (3)$$

Because of the existence of a cryogenic temperature region in the pulse tube cooler, a compressibility factor is considered in the description of the gas state. Hence, equation of state for gas is:

$$P = Z\rho RT \quad (4)$$

Energy equation for the solids:

$$\frac{\partial}{\partial t} [(1-\phi)\rho_s C_s T_s] = \frac{\partial}{\partial x} \left((1-\phi)k_s \frac{\partial T_s}{\partial x} \right) + \alpha A_V (T - T_s) + \dot{q}_{ext} \quad (5)$$

Last term of Eq. (5) is the external heat per unit volume that is loaded to the warm heat exchanger and the cold heat exchanger.

2.3. Boundary and initial conditions

No-penetration (zero velocity) condition must be satisfied at the four end boundaries (two ends of SWE and two ends of PTC).

Initial conditions include zero velocity, charge pressure, and the ambient temperature. Because of the large heat capacity of the solid matrix, it takes thermal behavior a long time to reach periodic (cyclic) steady state. Heat transfer area of the parallel plate stack is very smaller than that of the regenerator. Thus, the temperature of the parallel plates is the last parameter which will reach the thermal periodic steady state. Since only the periodic steady state solution is sought, the initial conditions for temperature are given as close as possible to the final solution.

The program developed in the present model can let the gas oscillate by itself in acoustic engine because of numerical noises produced in the calculation procedure. However, an initial small mass flow rate is given to the program to reach steady conditions sooner.

2.4. Empirical relations

It can be easily shown that the gas–solid heat transfer area per unit volume for each medium (tubular parts, parallel plate stack, and wire mesh screens) can be calculated as follows:

$$A_V = \frac{4\phi}{d_h} \quad (6)$$

Porosity for tubular parts is 1 and for other parts can be easily calculated by using geometrical relations. Hydraulic diameters are presented in Table 1.

The friction forces in the momentum equation are formulated based on porous medium characteristics and are generalized for all parts. For the parts that Darcy permeability and Forchheimer's inertial coefficient have not been defined, friction factor is related by using the following relation:

$$\frac{\partial p}{\partial x} \equiv \frac{f}{2d_h} \rho u |u| \equiv \left(\frac{\mu}{K} \phi u + \frac{C_F \rho}{\sqrt{K}} \phi^2 u |u| \right) \quad (7)$$

Table 1 presents empirical relations of friction factors for different geometries used in the construction of the device.

We notice that inside SWE, minor losses must be considered carefully as well as the major friction. Nevertheless, inside the pulse tube cooler, mentioned frictions are small in comparison to regenerator Darcy and Forchheimer's effects.

The heat transfer coefficient is defined as follows:

$$\alpha = \frac{kNu}{d_h} \quad (8)$$

Nusselt numbers for different parts are tabulated in Table 1. Reynolds number is:

$$Re = \frac{\rho |u| d_h}{\mu} \quad (9)$$

3. Numerical simulation

To solve the Eqs. (1)–(5), the computational domain is divided into several finite volumes, and then the equations are integrated

Table 1
Empirical relations used in the model.

Part	L (cm)	d (mm)	Solid medium
Hot buffer	5	50	Tube
Stack	6	50	SS parallel plates
Resonator	70	29	Tube
Ambient buffer	29	59	Tube
APAT	50	4.3	Copper tube
Regenerator	3	10	600# SS screen
Pulse tube	4	6	Copper tube
Inertance tube 1	30	2	Tube
Inertance tube 2	110	4.3	Tube

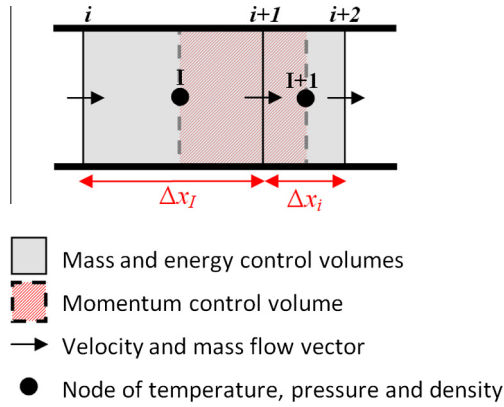


Fig. 2. Staggered grid used in the model.

over each corresponding volume (I -cv for continuity, energy and state equations and i -cv for momentum equation). A staggered grid is used for spatial discretization (Fig. 2). Integration of continuity equation over a control volume gives:

$$\frac{\partial m_I}{\partial t} + \dot{m}_{i+1} - \dot{m}_i = 0 \quad (10)$$

$$\begin{aligned} \frac{\partial (mu)_I}{\partial t} + \dot{m}_{i+1}u_{i+1} - \dot{m}_iu_i = & -A_i\phi_i(P_I - P_{I-1}) \\ & - \left(\frac{\mu}{\rho K} \phi + \frac{C_F}{\sqrt{K}} \phi^2 |u| \right)_i \Delta x_i \dot{m}_i \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left[m \left(C_v T + \frac{u^2}{2} \right) \right]_I + \dot{m}_{i+1} \left(C_p T + \frac{u^2}{2} \right)_{i+1} - \dot{m}_i \left(C_p T + \frac{u^2}{2} \right)_i \\ = \left(k \phi A \frac{\partial T}{\partial x} \right)_{i+1} - \left(k \phi A \frac{\partial T}{\partial x} \right)_i + \alpha_I A_{V,I} V_I (T_{s,I} - T_I) \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{\partial}{\partial t} [m_s C_s T_s]_I = \left(k_s A_s \frac{\partial T_s}{\partial x} \right)_{i+1} - \left(k_s A_s \frac{\partial T_s}{\partial x} \right)_i + \alpha_I A_{V,I} V_I (T_I \\ - T_{s,I}) + \dot{q}_{ext} V_I \end{aligned} \quad (13)$$

$$P_I V_I = Z_I m_I R T_I \quad (14)$$

The second order backward approximation is used for the time derivative except for first step calculations, that first order backward is used. The second order upwind scheme is used for the convective (the enthalpy flow) terms as well as for the momentum flow. The thermal conductive terms are approximated by central difference scheme. Equation of state is put into continuity equation, so a relation is approached for pressure as an explicit function of mass flow rates.

Finally, the equations are changed into algebraic form. The solving process is as follows:

1. Solving the momentum equation.
2. Calculation of the pressure, explicitly.
3. Solving the energy equation of the gas.
4. Solving the energy equation of the solid.
5. Repeating steps 1 to 4 until one time layer is finished.
6. Repeating steps 1 to 5 until the periodic steady state is achieved.

4. Results and discussion

4.1. Grid Independence, and validity of the model

To analyze the grid independence of the solution, a series of simulation runs have been carried out. In order to reduce the

CPU time in grid independence study, the simulations were performed for only 100 cycles. Several mesh densities were tested so that for each part of the system, the number of nodes was increased and the results were checked. For the stack, the number of nodes is more than that of the other parts, because the large temperature gradient of the stack makes larger false numerical diffusions in coarser grid. This will be sensible, especially in the calculation of work or energy flows. The cycle-averaged acoustic power at the engine outlet (APAT inlet) was calculated according to Eq. (15) for the 100th cycle. Fig. 3 shows the acoustic power for different number of grids. Note that the presented values of Fig. 3 are not steady values, because the flow field solution requires about 1000 cycles to reach steady periodic state. The calculated steady acoustic power at the engine outlet is 61.8 W. With the total number of nodes of about 350, it can be inferred that the results are independent of the grid system.

$$\dot{W}_{ac} = \oint (P u A) dt \quad (15)$$

In order to validate the model and method of solution, a comparison is made against the previously published experimental work of Zhu et al. [28]. The system, including stack 2, resonator, 1, and APAT 1 of their work is modeled. The characteristics of the system have been listed in Table 2. Input heating power is 1000 W that is loaded to the warm heat exchanger.

The time step size of 5.0×10^{-5} s (i.e. each cycle is divided into 66 steps, approximately), and a total node of 344 are used. For the parts with large temperature gradient such as stack, regenerator, and pulse tube, finer grid is used, whereas long parts such as APAT, resonator, and inertance tubes are discretized with coarse grid. Simulation run is performed for about 1000 cycles, so a steady periodic condition is reached. At each time step, 20 inner iterations are used for proper coupling between the equations and for better convergence. Running one simulation case by a computer with 2.4 GHz processor and 1.0 GB RAM until the steady state takes about 1.6 h.

Fig. 4 shows the cooling temperature versus cooling power of present results and the results reported by Zhu et al. [28]. The relation between cooling power and cooling temperature (so-called cooling curve) is often nearly linear and the slope of the cooling curve is almost equal for two results. The present cooling curve is almost identical to that of Zhu et al. but less than that. The simulated steady cycle-averaged no-load cooling temperature is 63.3 K, which is lower than the experimental result 72 K. It must be noticed that the thermal parameters, especially, the cooling power are the most complicated parameters that have large errors. For other parameters such as operating frequency, flow rates, and

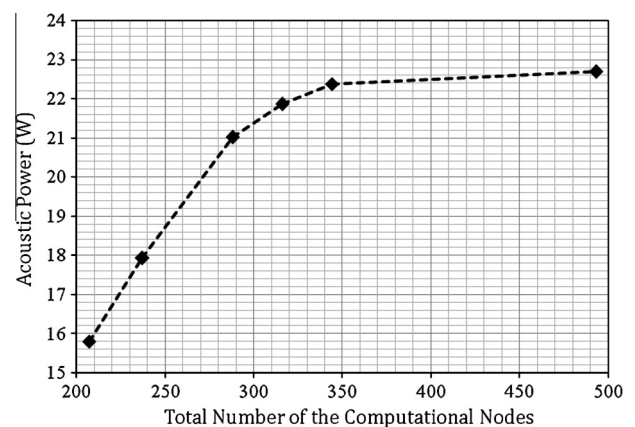


Fig. 3. Grid independence of the acoustic pressure.

Table 2
Characteristics of the modeled system.

Medium	d_h	Friction equation	Heat transfer relation
Tube	d	Laminar: Turbulent:	$Nu = 0.023Re^{0.8}Pr^{0.35}$
Wire screen	$\frac{\phi}{(1-\phi)}d_{wire}$	$\frac{\phi^2}{K} = \frac{33.6n\phi}{2l\beta} \frac{2\phi^3C_F}{\sqrt{K}} = \frac{0.337n\phi^2}{\beta^2}$	$Nu = (1 + 0.99(RePr)^{0.66})\phi^{1.76}$ [30]
Parallel plate	$2d_c$	Laminar: Turbulent:	$Nu = 1.3030Re^{0.3271}Pr^{1/3}$ [31]
		$f = \frac{96}{Re}$ $\frac{1}{\sqrt{f}} = 2\log_{10}(0.64Re\sqrt{f}) - 0.8$	

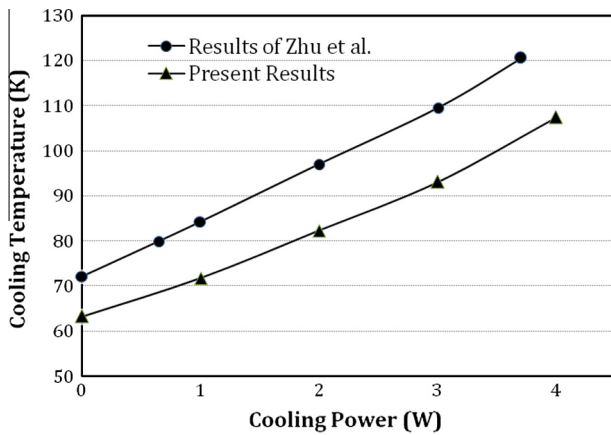


Fig. 4. Comparison of cooling temperature versus cooling power for present results and the results of Zhu et al. [11].

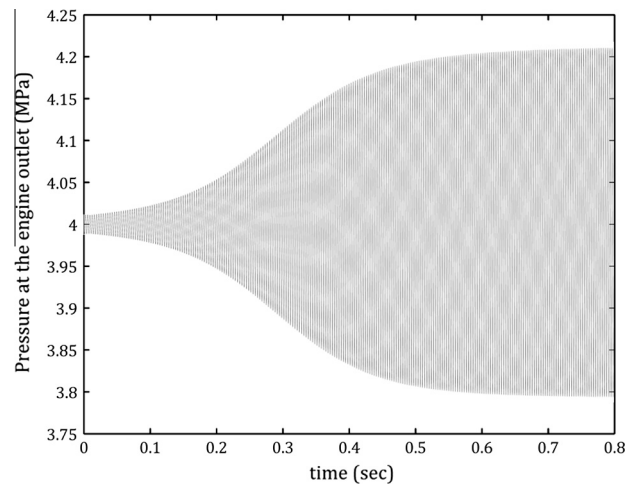


Fig. 5. The time history of the self-excited acoustic pressure oscillations.

pressure amplitudes, as presented in this paper, differences between the calculated and the experimental results are smaller. The main reasons of modeling errors are as follows:

1. Secondary flows inside pulse tube are the most important losses that result in a reduction of the net enthalpy flow from the cold end to the hot end of pulse tube. One-dimensional models cannot predict the secondary flows.
2. The empirical relations for friction factors and heat transfer coefficients have relative errors of about 10–15%.
3. Conduction heat transfer through the walls of regenerator and pulse tube from the hot ends to the cold end of them has not been considered in this study.
4. Radiation heat transfer from the hot ends of pulse tube and the solid matrix of regenerator to the cold end of them is a complicated phenomenon that has been neglected in the present work.

4.2. Nonlinear aspects

4.1.1. Self-excitation process

One of the most challenging purposes is to capture the evolution of the initial instability to steady state which the instability saturates to a finite amplitude. This is a strongly nonlinear effect and cannot possibly be traced by the models based on the linear theory. After transitioning from the initial velocity disturbance, the simulation yielded strong sustained pressure oscillations. Fig. 5 presents the time history of acoustic pressure at the engine outlet. The pressure amplitude tends to 0.21 MPa as it is reported previously [28]. An important capability of the present one-dimensional nonlinear model is precise tracing of the self-excitation process which a linear model has not it. Most of the previously reported time histories of pressure oscillations are obtained by CFD commercial softwares. It must be noticed that a smaller initial mass flow disturbance causes a longer time to reach steady periodic pressure oscillations. It was found that the flow field (pressure

and velocity) reaches the steady periodic condition, very faster than the thermal field does. The frequency which is calculated by the present numerical model is 303 Hz. It has an acceptable agreement with the experimental result of 300 Hz [28].

4.1.2. Steady state nonlinearities

After attaining the steady periodic state solution, we concentrate on steady nonlinearities of the results. Note that the linear model is based on a sinusoidal approximation of all variables and it is not permitted to be used when the pressure amplitude is greater than 10% of the mean pressure. For a general function $F(x,t)$ that represents the velocity, pressure, density, and temperature, Eq. (16) has been used for curve-fitting of the present results as functions of time and longitudinal coordinate. In most of nonlinear works, this form (up to a specified order) of unknowns was incorporated into the governing equations to derive x -dependent equations and to solve them.

$$F(x, t) = F_0(x) + \Re\{F_1(x)e^{j\omega t}\} + \Re\{F_2(x)e^{2j\omega t}\} + \Re\{F_3(x)e^{3j\omega t}\} + \dots \quad (16)$$

The gas temperature at the middle of the stack along with its first and second order harmonic approximations have been illustrated in Fig. 6 to investigate nonlinearity of the flow field inside the standing wave engine. It is obvious that even the first order approximation acceptably fits the calculated data. The distinction between the calculated results and the second order approximation is not clear. Inside the whole engine, the ratio of the pressure amplitude to the mean pressure is less than 6%. We investigate the other results inside the engine and conclude that the linear theory can predict oscillations satisfactorily, except a narrow region containing the minimum pressure amplitude point. As it is presented and discussed at the end of Section 4.3, some nonlinearities and jump to the second harmonic occur in this region of the resonator.

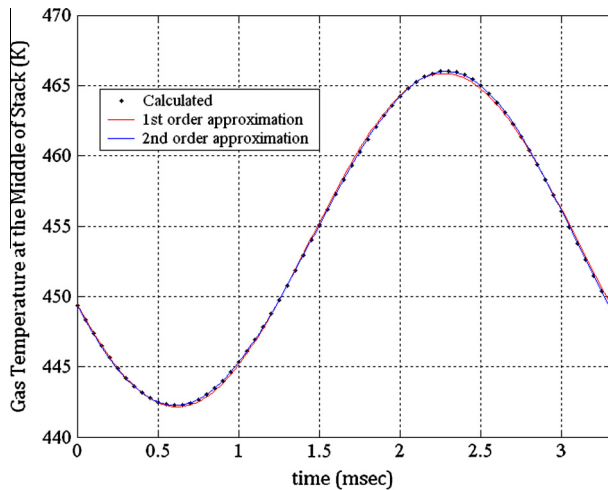


Fig. 6. The gas temperature at the middle of the stack versus time and its harmonic approximations.

Inside APAT and the cooler, high amplitude oscillations occur. The results indicate that inside the cooler, the oscillations deviate from linear theory, because, the pressure amplitude is amplified by APAT so that it exceeds from 10% of mean pressure. The most important sources of nonlinearities are friction forces, including minor losses and Forchheimer's inertia, and advective acceleration (in sections with very large diameter changes). Here, we present only two selected parameters: the mass flow rate at the cooler inlet and the gas temperature at the cold end of pulse tube.

Fig. 7 shows the mass flow rate at the cooler inlet along with its first, second and third order approximations. First and second order approximations can not predict the mass flow rate oscillations precisely. It can be inferred from Fig. 7 that the mass flow rate oscillates by third order harmonic. It is noteworthy that based on the conservation of mass, the cycle-averaged mass flow rate at any cross section must be zero, but due to the numerical errors, this criterion has never been satisfied and it tends to a very little steady value.

The deviations are more obvious when the temperature field inside the cooler is investigated. The calculated results and the approximations (up to 5th order) of the gas temperature at the cold end are illustrated in Fig. 8. For better distinction, only first, third, and fifth order approximations are presented. The results show

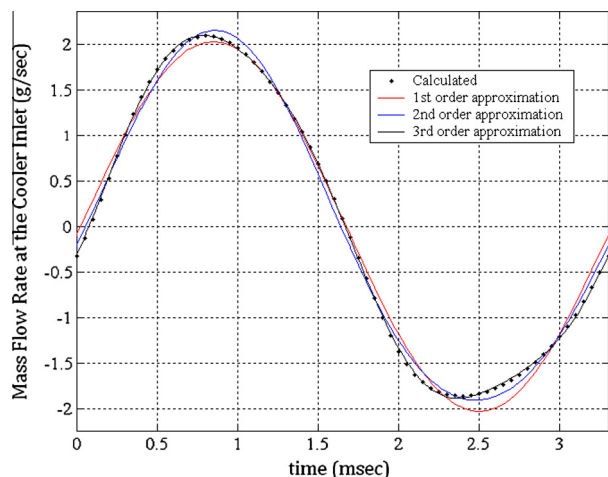


Fig. 7. The mass flow rate at the cooler inlet versus time and its harmonic approximations.

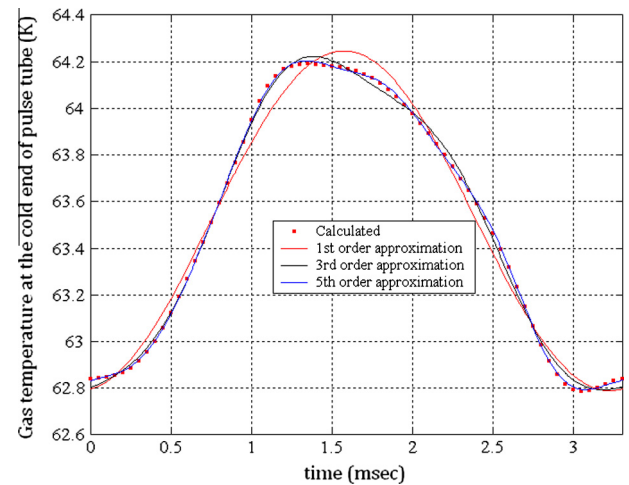


Fig. 8. The gas temperature at the cold end of pulse tube versus time and its harmonic approximations.

that a high order nonlinear thermoacoustic is necessary to model the temperature field of the gas inside the cooler.

Hence, inside pulse tube, because of the high-pressure amplitude amplified by APAT, oscillations of the flow and thermal field are not sinusoidal, especially the temperature variations, so that implementing a nonlinear model is essential.

4.3. Steady periodic state

After the steady periodic condition is reached, we will concentrate on one-cycle oscillations. To illustrate the order of magnitude of the gas that flows to the engine and to the cooler, Fig. 9 shows mass flow rates at branch faces at the moment of mean pressure. The magnitudes are close to maximum magnitudes of cyclic mass flow rates. Inequality of mass flow rates into branch control volume causes a change in density of gas.

Parametric study of APAT gives useful knowledge to optimize and investigate the flow phenomena inside the system. Inertial effects, pressure drop, and mass storage are the dimension dependent parameters that affect the amplitude and phase of pressure and mass flow rate, and the performance of pulse tube cryocooler subsequently. In order to study the influence of the APAT on the parameters, simulations were done by using an APAT with inner diameter of 4.3 mm and several different lengths. Curve fitting of the mass flow rate, velocity, temperature, and pressure has indicated that the oscillations are about sinusoidal inside the engine, but inside the cooler, deviations from sinusoidal behavior has been seen, especially in the temperature oscillations.

APAT changes amplitude and phase of oscillating parameters while gas is flowing through it. Simultaneously, it decreases acoustic power a little. Variations of mass flow rate at the engine outlet and the cooler inlet versus time for different APAT lengths are shown in Fig. 10. As APAT length increases, amplitude of mass flow rate increases. The ratio of mass flow rate at the engine outlet to mass flow rate at the cooler inlet increases as APAT length increases. APAT induces a phase lag nearly proportional to its

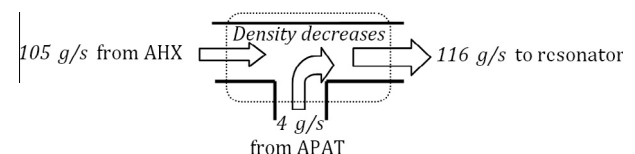


Fig. 9. Mass flow rates in branch at the moment of mean pressure.

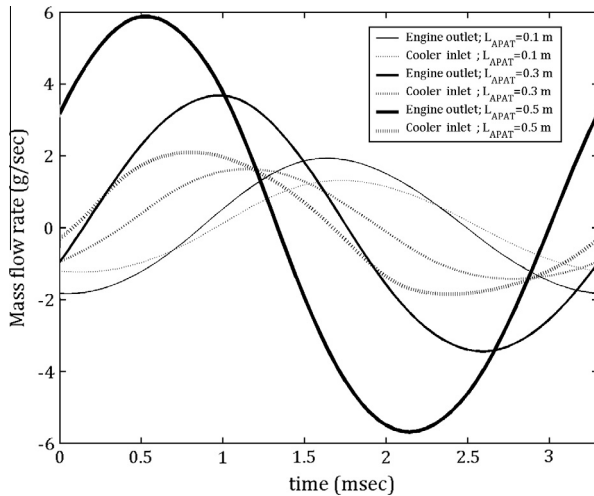


Fig. 10. Mass flow rate versus time for different APAT lengths.

length. The phase difference between mass flow rates of the engine outlet and cooler outlet is 7.6 deg for $L_{APAT} = 0.1$ m, 19.7 deg for $L_{APAT} = 0.3$ m, and 44.0 deg for $L_{APAT} = 0.5$ m.

Fig. 11 shows pressure oscillations for different APAT lengths. It is clear that a suitable APAT length can really lead to a more intense acoustic pressure wave available to the cooler than that provided by the engine. Increasing of APAT length has not significant effect on the amplitude of the pressure at the engine outlet so its value remains constant at the value of 0.21 MPa, approximately. Nevertheless, the pressure amplitude at the cooler inlet and phase difference of the pressure wave at two ends of APAT, increase as APAT length increases. The phase difference between the pressure at the engine outlet and pressure at the cooler inlet for APAT lengths of 0.1, 0.2, 0.3, 0.4, 0.5, and 0.6 is 32.70 deg, 27.80 deg, 25.10 deg, 18.50 deg, 15.30 deg, and 0.90 deg, respectively. For better distinction, not all graphs of different simulated APAT lengths have been depicted in Fig. 10 and Fig. 11.

For APAT lengths of 0.1, 0.2, 0.3, 0.4, 0.5, and 0.6, phase differences between the mass flow rate and pressure at the cooler inlet are 65.5°, 58.9°, 53.5°, 50.2°, 50.2°, and 50.2°, respectively, Whereas it is 27.3 deg for a typical compressor driven pulse tube cooler.

APAT dimensions affect the flow field of the engine domain as well as flow field inside itself and the cooler. As it is illustrated in Fig. 12, volume flow rate inside the engine is influenced by APAT

length. The volume flow rate inside the engine has a maximum value at the position of the resonator where the minimum pressure amplitude occurs, at $x = 0.2$ m from left end of the resonator. Inside the ambient buffer, the volume flow rate tends to zero linearly by steep slopes. Increasing of APAT length causes a larger amount of gas flows into PTC, thus the volume flow rate along SWE decreases a little.

Three phenomena dominate the pressure field:

- (1) Amplification (inside APAT): The pressure amplitude increases.
- (2) Resonating (inside the resonator and inertance tube): a phase shift of pressure oscillation is produced to make the proper volume flow rate.
- (3) Friction (All parts; especially in porous media): The pressure amplitude decreases especially inside the regenerator.

The distribution of the maximum and minimum of the pressure oscillation along the device are illustrated in Fig. 13. Inside tube parts such as hot buffer and pulse tube, there is a little change of magnitude of pressure amplitude. Inside APAT, the pressure amplitude increases and the velocity amplitude decreases. We can divide the resonator (also inertance tube) into three regions: left that includes about two third of the resonator length, mid-region that is a narrow region about a few centimeters, and right that about one third of the resonator length. In left region, the pressure amplitude decreases from the left (inlet) to the lowest pressure amplitude point, while the pressure phase does not vary so much. In the mid-region where the lowest pressure amplitude point is located, the pressure phase changes about 180° (see Fig. 14). This phase change is not provided by lagging but rather by jump from base harmonics to second harmonics. At $x/L = 0.7$ frequency is twice as large as the base frequency. The change of harmonics in Fig. 14 is evident.

From the minimum pressure amplitude point to the right end (outlet), the pressure amplitude increases, while the pressure phase does not vary so much.

4.4. Total performance

From the previous experiences, a great performance could be easily obtained by using an APAT. Calculations indicate that a longer APAT length gives a better acoustic performance, but because of the safety requirements (the highest hot temperature of 923 K in the present case), it is limited to a maximum value.

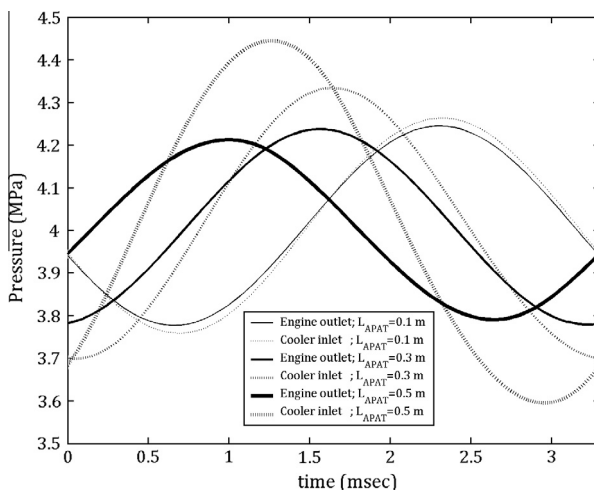


Fig. 11. Pressure oscillations for different APAT lengths.

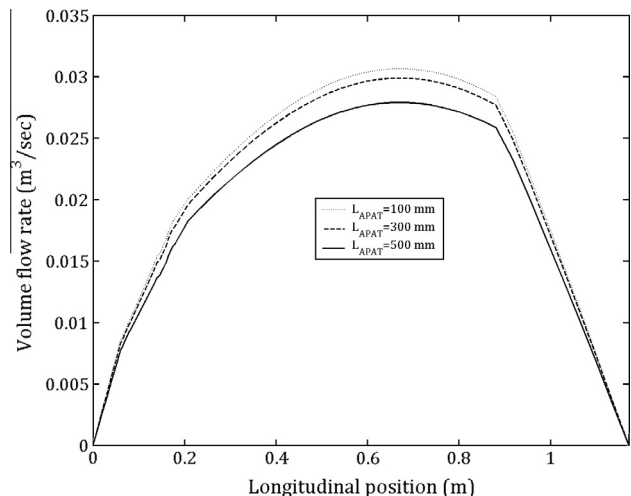


Fig. 12. Amplitude of volume flow rate along SWE for different APAT lengths.

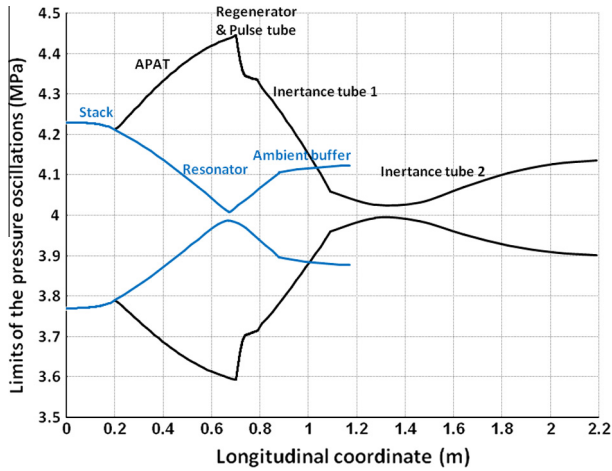


Fig. 13. The distribution of pressure oscillation limits along the device.

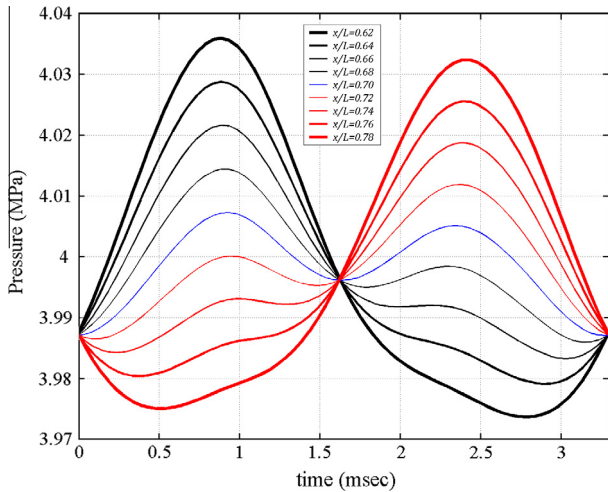


Fig. 14. Pressure oscillations during one cycle at different (normalized) locations inside the resonator.

Hot heat exchanger temperature for different APAT lengths is calculated and shown in Fig. 15. The experimental results are also presented for comparison. Obviously, when APAT length exceeds 600 mm, highest temperature increases over 923 K. The difference between the numerical and reference values are less than 20 K, hence, an acceptable agreements are reached.

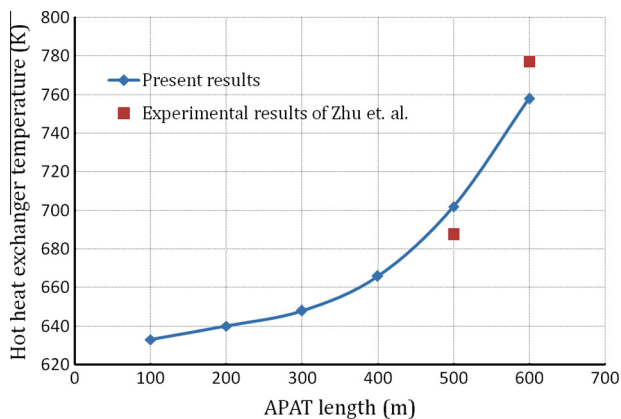


Fig. 15. Hot heat exchanger temperature versus APAT length.

The pressure ratio is defined as the ratio of the maximum pressure to the minimum pressure at the cooler inlet. The no-load cooling temperature and the pressure ratio are shown in Fig. 16. It must be mentioned that the graph of the pressure ratio is in good agreement with the experimental results except for small APAT lengths, because it must tend to 1.0 in the case that no APAT is used. It is obvious that there is an inverse relation between no-load cooling temperature and the pressure ratio.

4.5. An extra challenge: Lagrangian approach

By using the values of last calculated cycle in Eulerian coordinate, an additional Lagrangian approach is used to track the movement of different gas parcels to get their pressure, temperature, and velocity in a thermodynamic cycle. For the first time level, we fix Lagrangian coordinate (location of the particles) to Eulerian coordinate. Thus, all variables are same in the two coordinates. The gas parcel will move to a new position at the new time level as it oscillates. Assuming that the velocity of the gas parcel is constant during a time step, the new location of the gas parcel can be easily calculated. With the assumption that values are linear between two adjacent nodes, if the new position is not located in a grid point, pressure, temperature, and velocity of the gas parcel are calculated by linear interpolation of those parameters at two adjacent grids. After 66 time steps, the gas parcel will return to its original position at the beginning of the cycle. Therefore, thermodynamic cycles of the gas parcel can be obtained. Results show that different gas parcels undergo thermodynamic cycles at different temperatures. In this section, Lagrangian approach results of modeling of the device with specifications listed in Table 2 are presented.

For thermoacoustic engines, thermodynamic cycle of gas parcels inside the stack is an elliptical path as seen from u - x diagram shown in Fig. 17 as well as T - x diagrams shown in Fig. 20. Fig. 17 shows the velocity of three chosen gas parcels that are located in the stack. It is clear that the elements that are close to the cold end of the stack have more displacement and velocity variations than those are close to the hot end. The displacement of gas parcels is in order of 0.01 m.

The acoustic power is proportional to the multiplication of pressure and velocity. APAT wastes the acoustic power due to wall friction. The pressure increases along APAT, and velocity is decreased along it. It means that the gas parcels near the cooler inlet (APAT outlet) side have less velocity amplitude than those near the engine outlet as shown in Fig. 18. The gas parcels near the engine outlet, and near the cooler inlet have a velocity magnitude of 60 m/s and 25 m/s. It implies that the linear thermoacoustic model cannot be used because of large inertial effects. The velocity varies linearly inside APAT. It can be seen from Fig. 17 that the gas parcels closer to the cooler inlet have less displacement. The maximum

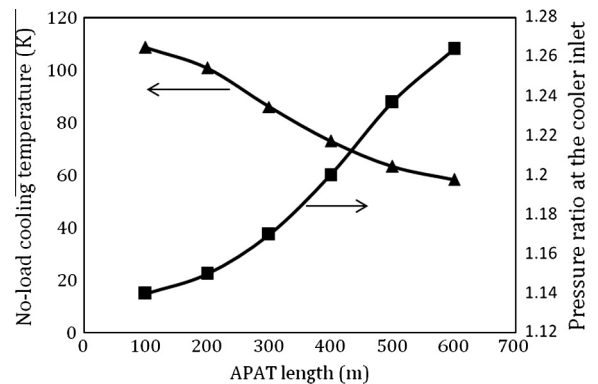


Fig. 16. No-load cooling temperature & pressure ratio versus APAT length.

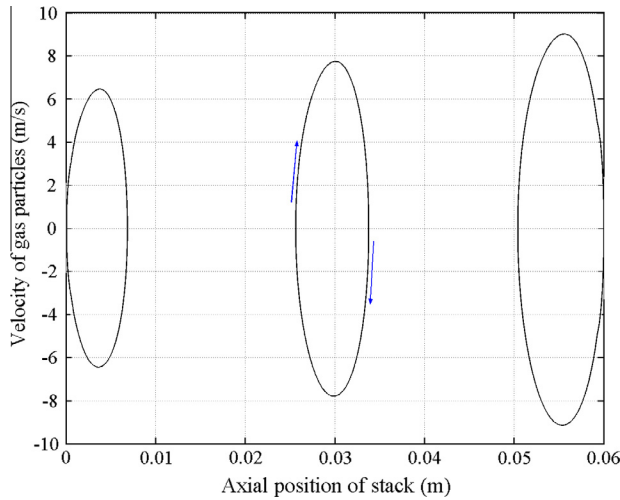


Fig. 17. Velocity of gas parcels inside the stack during one thermodynamic cycle.

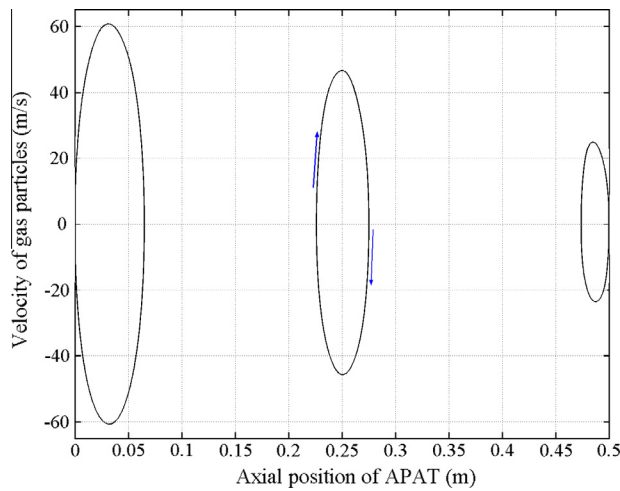


Fig. 18. Velocity of gas parcels inside APAT during one thermodynamic cycle.

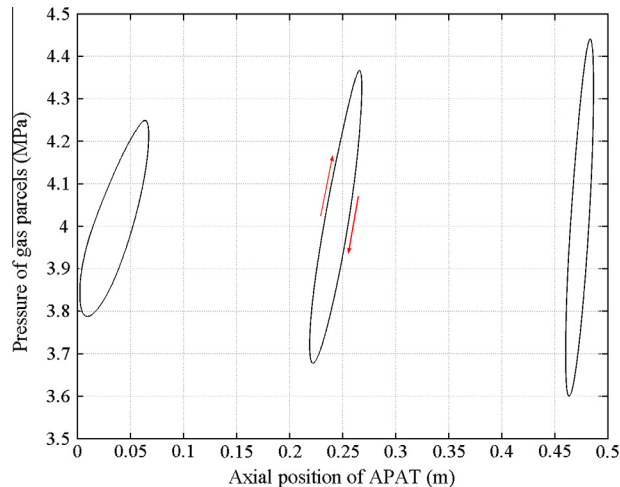


Fig. 19. Pressure of gas parcels inside APAT during one thermodynamic cycle.

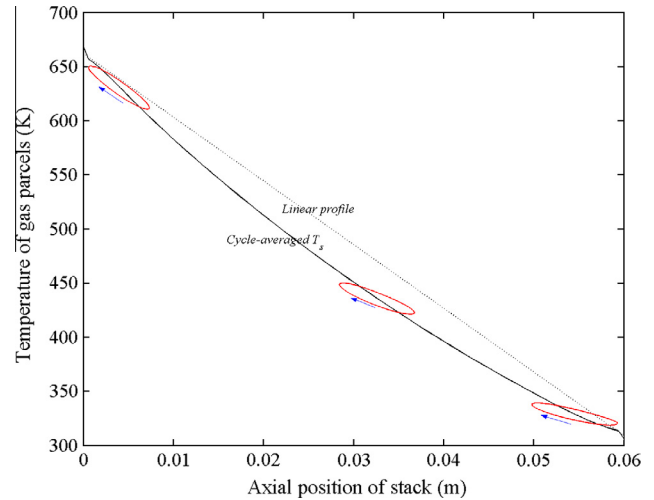


Fig. 20. Temperature of gas parcels inside the stack during one thermodynamic cycle.

reciprocating compressor for compressor-driven pulse tube cryocooler. Hence, its velocity is equivalent to the compressor frequency and its displacement is equivalent to stroke of the piston.

Fig. 19 shows the pressure of three gas parcels inside APAT. The clockwise thermodynamic path means positive mechanical work (work input). The input power is equal to the area of the P - V cycle. Again, we can see that the gas parcels near the cooler inlet side have shorter displacement than those are near the engine outlet side of APAT. Along APAT, P - V diagrams become narrower, and the pressure amplitude becomes larger. It can be seen that the P - V diagram in the APAT inlet is tilted to the right largely, whereas gas parcels near the cooler inlet have a nearly symmetric path.

Fundamentally, thermoacoustic devices can be either engines or refrigerators. It depends on direction of gas-wall heat transfer inside the stack in a standing wave device (or regenerator in a traveling wave device). For standing-wave devices, the direction of the heat transfer between the gas parcels and the solid surface near each end of the stack is decided by the critical temperature gradient.

Fig. 20 shows the temperature of three local gas parcels as they oscillate along the stack. The cycle-averaged temperature of solid plates is also presented. For clarity, virtual linear temperature distribution is shown. The cycle-averaged temperature of the gas (in Eulerian coordinate) is equal to that of solid plates, approximately. They can be estimated well with the following power function:

$$T = T_H \left(\frac{T_C}{T_H} \right)^{\frac{x}{L}} \quad (17)$$

Here, x is the longitudinal position from the hot end of the stack and L is the length of the stack. T_C and T_H are the cold end and the hot end temperatures of the stack, respectively.

The clockwise direction of thermodynamic cycles is also illustrated in Fig. 20. It can be clearly seen that the stack solid surface temperature is higher than the gas parcel near the hot end of thermodynamic path, while the opposite is true near the cold end. The temperature of the parcel traces out a clockwise ellipse as a function of time.

As a gas parcel oscillates along the axis of the parallel plates, it experiences changes in temperature, caused by adiabatic compression and expansion produced by the sound pressure and by gas-wall heat exchange. The gas parcels move to the hot side of stack while the pressure is rising. In this moment, the gas parcel absorbs heat from the adjacent plate wall. Then, it moves to the

and minimum displacements of gas parcels inside APAT are 65 mm and 26 mm at the hot and cold ends of APAT, respectively. The gas parcel that is next to the cooler inlet, acts as piston of a

cold side while the pressure is falling. Now, the gas parcel rejects heat to the plate. Imperfect gas–wall heat exchange is required in order to provide a time delay between the gas motion and gas thermal expansion or contraction. It results when the plate spacing is more than thermal penetration depth.

5. Conclusions

A nonlinear one-dimensional model for modeling of the flow and thermal field of a typical high frequency thermoacoustically driven pulse tube cryocooler has been developed. The finite volume method has been used for discretization of governing equations. The self-excitation process has been simulated very well. Results show that the present model has an acceptable accuracy in calculation of flow characteristics of the system. The effects of APAT on the performance of the device have been investigated. As an extra challenge, Lagrangian approach has been implemented to trace the thermodynamic cycle of the gas parcels.

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